



全国水利行业规划教材

“十三五”江苏省高等学校重点教材(编号:2019-1-100)

Coastal Hydrodynamics
and
Morphodynamics

海岸动力学

· 港口航道与海岸工程专业 ·

The 2nd Edition

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人民交通出版社股份有限公司

北京

内 容 提 要

“海岸动力学”是港口航道与海岸工程专业的核心基础课程。为适应双语教学的需要,根据国内教学大纲,在河海大学主编的《海岸动力学》(英文版)(第1版)的基础上,进行再版更新。

本书共分8章,内容包括绪论、波浪理论、随机波浪、波浪传播变形、近岸流、无粘性泥沙运动、粘性泥沙运动和海岸地貌演变,每章后附有相关知识的拓展阅读内容、重要专业词汇的英汉对照表、链接线上资源的二维码和习题。

本书入选全国水利行业规划教材和“十三五”江苏省高等学校重点教材,是国家级精品资源共享课和国家级线下一流本科课程的一个重要组成部分,曾获徐芝纶教材奖二等奖,可供高等院校港口航道与海岸工程、船舶与海洋工程、海洋资源开发技术、土木工程、环境工程以及其他相近专业开设海岸动力学双语课程使用,也可供相关领域涉外工程技术人员使用。

图书在版编目(CIP)数据

海岸动力学:英文/张弛,张继生,郑金海主编
—2版.—北京:人民交通出版社股份有限公司,
2022.8

ISBN 978-7-114-18148-1

I. ①海… II. ①张… ②张… ③郑… III. ①海岸—
海洋动力学—高等学校—教材—英文 IV. ①P731.2

中国版本图书馆 CIP 数据核字(2022)第 147432 号

书 名: **Coastal Hydrodynamics and Morphodynamics**(The 2nd Edition)

海岸动力学(第2版)

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责任编辑: 黄兴娜

责任校对: 赵媛媛

责任印制: 刘高彤

出版发行: 人民交通出版社股份有限公司

地 址: (100011)北京市朝阳区安定门外外馆斜街3号

网 址: <http://www.chinasybook.com>

销售电话: (010)64981400, 59757915

总 经 销: 北京交实文化发展有限公司

印 刷: 北京虎彩文化传播有限公司

开 本: 787×1092 1/16

印 张: 12.75

字 数: 393千

版 次: 2015年9月 第1版

2022年8月 第2版

印 次: 2022年8月 第2版 第1次印刷 累计第3次印刷

书 号: ISBN 978-7-114-18148-1

印 数: 0001~1000册

定 价: 50.00元

(有印刷、装订质量问题的图书由本公司负责调换)

► Preface

Coastal hydrodynamics and morphodynamics is an important course for undergraduates majoring in Harbor, Waterway, Coastal and Ocean engineering. The first textbook of coastal dynamics in Chinese was published in 1980. For the purposes of bilingual teaching to serve internationalized talents training, the first textbook of coastal dynamics of English version which applies to China's national syllabus was published in 2015. The present textbook is the second-edition of the English textbook.

Chapter 1 gives a general introduction to coastal dynamics. Chapter 2 presents some fundamental wave theories that emphasises on the small-amplitude linear wave theory which is relatively simple and useful. Waves observed in the ocean are highly irregular and Chapter 3 is devoted to the analysis of random waves, which provides information for the design and maintenance of coastal and ocean structures. Wave transformation in the intermediate and shallow waters is introduced in Chapter 4, including wave shoaling, attenuation, refraction, diffraction, reflection and breaking. Chapter 5 presents nearshore currents with detailed information on the wave-induced currents. The characteristics and transport of noncohesive sediment are given in Chapter 6. Cohesive sediment or mud is encountered in many coastal areas and is described in Chapter 7, in which the focus is on the behavior and modeling of cohesive sediment motion. Chapter 8 presents coastal morphological processes on different scales and their response to engineering structures.

In this second-edition textbook, new materials have been added and updated according to the most recent progresses in scientific research and real-world engineering practices. In particular, the new concepts and methods (e. g. , nature-based solutions for disaster mitigation, beach safety knowledge) involved in the state-of-the-art coastal engineering and management have been added. The implications of coastal dynamics to China's coastal development as well as the important advances have been introduced. Several examples of representative engineering applications (e. g. , beach nourishment project, shoreline protection at Friendship Port in Mauritania) at home and abroad have been introduced. At the end of each chapter, the extended reading, quick response codes linking to the online resources, and exercises have been added. Chapters 1-2 were written by Zheng Jinhai at Hohai University, Chapters 3-4 were written by Zhang Jisheng at Hohai University, Chapters 5-8 were written by Zhang Chi at Hohai University.

This book is suitable as a textbook for undergraduate students related to Harbor and Waterway Engineering, Coastal and Offshore Engineering, Naval Architecture and Ocean engineering, Marine Resources Development Technology, and Civil and Environmental Engineering. It also serves a useful reference for scientific research and engineering works in the related fields.

The compilation, revision and publication of this textbook was supported by the Key Textbook Project for Institutions of Higher Learning of Jiangsu Province (No. 2019-1-100).

Author
April, 2022

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Chapter 1

Introduction

1.1 Foundation of Coastal Dynamics

Coastal dynamics is a subject of systematic scientific research with a long history, and a professional fundamental module in harbor, waterway, coastal and ocean engineering. It mainly includes the dominant coastal hydrodynamic processes (e. g. , wave, current) and the coastal morphodynamic processes (e. g. , sediment transport, bathymetry and shoreline evolution), as well as their interactions with practices of coastal engineering construction, coastal resources utilization, coastal protection and restoration, coastal disaster prevention and mitigation, and sustainable coastal management under the changing climate and human activities.

Due to the gradual shortage of land resources, human activities are extended to the ocean, and the economic status of the ocean has been rapidly accelerated. Coasts are important to our lives because of their close relationship with human beings. Since the 1920s, the Netherland and the United States began to pay attention to the research of coastal engineering and coastal dynamics. After decades of development, significant progresses have been made. This has also aroused the attention of many maritime countries to the coastal and marine resources exploitation and water transportation. Since the 1990s, the marine economy has stepped into the fast lane of world's economic development. In the 21st century, some emerging significant economic areas, for example, coastal tourism and recreation, have represented new growth points of marine economy for many nations. Therefore, the phenomena and processes that influence these activities in coastal regions should be understood.

Coastal processes, e. g. , waves, currents, and sediment transport should be analyzed and reliably predicted to provide necessary information for the appropriate design and implementation of coastal engineering projects. These projects include survey, planning, design, construction, maintenance and upgrade for engineering of various types, such as shoreline protection and restoration, dredging for deepening harbors and waterways including the suitable disposal of sediment, the rehabilitation and improvement of harbors including deep-water ports, navigation channels and waterways, as well as navigation safety and aids.

The impacts of coastal hydrodynamics and morphodynamics on the engineering are described by many observable phenomena, such as wave growth, transformation, wave set-up, run-up, overtopping, wave-induced current, nearshore circulation, sediment transport, beach profile and shoreline evolution. On the other hand, the effects of the engineering projects on the coastal dynamics should also be studied. These effects include the short-term and long-term morphology changes resulted from man-made hard structures and other activities that interfere with the antecedent dynamic environment or lead to the change of coastal sediment supply. When natural or human-induced changes occur, a new set of forces is induced to re-establish a dynamic equilibrium state which adapts to these changes. The above processes can be further complicated by the sea level rise, potentially intensified storm events and other impacts due to climate change acting on coasts globally or regionally.

Physical models, numerical models and field surveys are widely used in the study of coastal dynamics. A physical model is often made by scaling down the real coastal processes into the laboratory, according to the principle of similarity. A numerical model is a kind of mathematical method that is expressed by governing theoretical equations or empirical relationships, and is solved with numerical schemes using computers. Physical and numerical modelers shall be skeptical about their results and continuously compare them with well-documented site studies and field observation. For many cases, the knowledge and experience gained from a variety of field observations and real-world engineering projects is appreciable to improve our fundamental understandings of coastal dynamics, and hence to improve the prediction reliability of physical and numerical models.

The objective of this book is to provide some basic knowledge to assist students majoring in harbor, waterway, coastal and ocean engineering or in other related fields, to appropriately interpret complex coastal phenomena, predict the consequences of engineering construction, transform this knowledge into the effective measures required in various stages of real engineering projects, and guide manager decisions. Specifically, given changing weather conditions, bathymetry and sediment characteristics, we would like to analyze and predict how waves could change, and how those changes could affect currents and the erosion or accretion of the morphological elements. A better understanding of nearshore waves, currents, sediment transport, morphological evolution is crucial to develop strategies and tools to achieve a maximum benefit of coastal engineering practice, sustainable resources utilization, effective disaster mitigation and enhanced coastal resilience.

1.2 Practical Implication of Coastal Dynamics in China

China has rich coastal and marine resources. The resources in the coastal zone are valuable for maritime transportation, aquaculture, tourism and recreation, oil extraction and mining, renewable energy utilization, and other undertakings which are of great significance to the national economic

and social development. China has a long history of coastal and marine development. Especially in recent years, the annual growth rate of the marine economy has been developing rapidly. This is demonstrated mainly as follows: the scope of marine activities expands in multiple directions, the economic aggregate increases rapidly, the economic growth rate is larger than the national economic growth rate in which the developed coastal areas make a leading contribution. In March 2021, the Ministry of Natural Resources issued China Marine Economy Statistics Bulletin 2020. According to this bulletin, the marine gross economic product of China in 2020 was 8001 billion yuan, accounting for 7.9% of the national gross domestic product and 14.9% of the coastal areas' gross economic product. With the continuous growth of China's economy and the strengthening of the China's Maritime Power Strategy, the status and contribution of the marine economy in the national economy will be increasingly enhanced in the near future.

In the recent decades, China has made remarkable achievements in coastal protection and restoration, estuarine regulation, port construction, cross-sea bridges and tunnels, islands and reefs development, marine renewable energy, ship repair and building, fishing industry, and integrated coastal management. The knowledge of coastal dynamics is closely related to these achievements.

(1) Coastal dynamics is important for coastal protection and restoration

China has a long mainland coastline of about 18000 km. Due to the combined effects of sea level rise, storm events, engineering construction, sediment extraction and the decreasing sediment supply from rivers, however, about 49.5% of the sandy coasts in China is suffering beach erosion, and the wetland area has decreased in many regions. This further causes negative impacts to the coastal environment, e. g. , the loss of natural functionalities and landscapes, the degradation of ecosystem, as well as the increasing risks of coastal hazard. On the other hand, there is an increasing demanding of coastal tourism and public recreation. At present, 47% of the China's marine economy growth is contributed by the coastal tourism. To fulfill the sustainable coastal economic and social development as well as the higher requirements of people's living quality, coastal protection and restoration engineering have been attached with great importance by the government and public under the national strategy of Ecological Civilization. During 2016-2020, about 1200 km long of coastline and 230 km² of wetland area have been restored in China.

Works of coastal protection and restoration can be divided into hard engineering and soft engineering. Hard engineering refers to the traditional approaches of building groins, seawalls, breakwaters and other hard structures near the coastline. While these hard engineering provide effective protection to upland properties, they may show adverse environmental effects or cause erosion on adjacent shores. Soft engineering refers to the environment-friendly and flexible approaches such as beach nourishment, vegetation planting and ecological rehabilitation of coastal structures. These soft engineering utilize the functionalities of natural elements to achieve a synergistic benefit of coastal erosion control, disaster mitigation and eco-environmental restoration, a concept of so-called Nature-Based Solutions (NBS). While there is a shifting paradigm from hard

engineering to the NBS for coastal protection and restoration in China and other countries, further fundamental research and technology development of this field are still demanded and many questions remain unanswered. In 2021, the Ministry of Natural Resources of China issued 20 English-version related technical guidelines (Technical Guidelines for Investigation and Assessment of Coastal Ecosystem, Technical Guidelines on Coastal Ecological Rehabilitation for Hazard Mitigation), translated by the National Marine Hazard Mitigation Service and Hohai University, in order to promote the internationalization processes of Chinese standards, techniques and experience, and to support the future research and development collaboration.

The choices of using hard or soft works depend on many factors affecting their feasibility and compatibility with respect to the site-specific features of coastal environment. No matter what choice is made, this should be supported by a good knowledge of coastal dynamics. For example, the offshore wave forcing and nearshore wave characteristics are important for accessing the dynamic adaptability, supporting structure stability and disaster mitigation performance of engineering works. The mechanisms of coastal sediment transport and morphological evolution are important for designing optimum engineering schemes to avoid mass sediment loss and enhance morphology stability under storm impacts and sea level rise. The nearshore flow circulation pattern is important for improving water quality and beach safety. In addition, all of the above coastal hydrodynamic and morphodynamic processes are tightly interacting with the eco-environmental response, which still requires large efforts of multi-disciplinary research.

(2) Coastal dynamics is important for maritime transportation

In order to adapt to the development trend of large ships in the world, research and construction are carried out for large ports and deep waterways. The input, output and throughput of China's coastal ports have increased continuously. In addition, great progresses have been made in the scientific research and engineering technology of deep-water harbor construction. The water depth of the outer channel of Shishi Gate of Ningbo Zhoushan Port has reached 22 m, and ships of 300,000 tons can enter and leave the port with full loads. The extension of the 12.5m deep-water channel of the Yangtze Estuary to Nanjing greatly promotes the river-sea combined transport. It is expected that in the future, China will build a considerable number of docks and waterways with berths of 200,000 to 300,000 tons or even 500,000 tons. Since 2016, the Ministry of Transport of China has issued a series of English-version professional standards of water transportation, including Code for General Design of Sea Ports, which have been used in the engineering projects at home and abroad.

Cross-sea bridge and tunnel construction has emerged as a hot point in China. For example, the Hong Kong-Zhuhai-Macao Bridge of 55 km long opened in October 2018, stands as the longest cross-sea bridge in the world. It is possible that more cross-sea bridges and tunnels may be built between the channel and the islands. Qiongzhou Strait is one of the three major straits in China. It is about 80 km long, 20-40 km wide, with an average water depth of 44 m and a maximum depth of

120 m. The planned Qiongzhou Strait cross-sea project will face challenges from deep water, strong wind, high wave, complex geological structure, high navigation requirements, sensitive environment and so on.

Since the future constructions of ports, cross-sea bridges and tunnels and other infrastructures of water transportation in the 21st-Century Maritime Silk Route are often located in quite unfamiliar sea areas, there is a significant need to study the site-specific features of coastal waves, currents, sediment transport and bathymetry evolution, which can be considerably different from or more complex than our past experience. This is important for increasing structure stability under wave and current impacts, decreasing sediment siltation in harbor basin and channel, suppressing the local scour around the structure foundation, and minimizing the effects on the adjacent coastal morphodynamic system.

(3) Coastal dynamics is important for islands and reefs development

China has a long island coastline of about 14000 km long, distributed with abundant natural resources, ecosystem and landscapes. Increasing attentions have been paid to the sustainable development and protection of island and reef coasts. For example, the development and protection of coral islands and reefs have been reasonably carried out in the past decade, in order to improve the coral ecosystem, restore the reef structure, protect the land resources, promote the tourism, and increase the disaster mitigation ability. This becomes more important in face of the increasing requirements of engineering works and the increasing threat of global climate change.

Coral island and reef coasts often have unique features of morphological system. These can be summarized by its specific location and platform, steep fore-reef slope, wide reef-flat with shallow depth, very large reef-flat bottom roughness, coral sand transport and appreciable alongshore morphology variability. This leads to quite different hydrodynamic features, including the intense wave breaking near the reef edge, a surf zone far from shoreline, remarkable friction dissipation, increasing importance of infragravity waves and wave setup, and complex circulation in the reef-lagoon-channel system. Nevertheless, our understanding of coastal hydrodynamics and morphodynamics of coral islands and reefs is much less when compared to other types of coasts.

Studying the above coastal dynamic processes of coral islands and reefs has many practical implications. For example, a better understanding of wave characteristics is important for designing coastal protection structures and increasing disaster mitigation; the pattern of flow circulation in the reef-lagoon-channel system is important for nutrients transport and pollution control; the mechanism of coral sand transport is important for maintaining the water depth of harbor and channel; the analysis of morphological evolution is important for the health and stability of coral ecosystem and reef structure, as well as for the survival of coral islands and reefs in response to sea level rise.

(4) Coastal dynamics is important for the marine renewable energy development

The development of marine renewable energy (e. g., wave energy, tidal energy, current

energy, wind energy and others) has quickly become a global hotspot. China's marine renewable energy development and utilization is still ongoing, but some achievements have been made including the demonstration project of Zhoushan tidal current energy finished in 2020, the offshore wind farm of Donghai Bridge finished in 2009, Jiangxia tidal experimental hydropower station finished in 1980, and others. At present, marine renewable energy utilization still cannot compete with conventional energy use. This is not compatible with the marine energy situation of China and the growing demand for clean energy. Putting more efforts into the marine renewable energy development is also helpful to achieving the national goals of Carbon Emission Peak by 2030 and Carbon Neutrality by 2060.

Many of the currently explored marine renewable energy comes directly from the hydrodynamic energy, while the seabed stability serves as an important morphological factor. Depending on the types of marine renewable energy, the energy conversion devices of different kinds are built at various water depths, some of which are located in intermediate or even shallow waters near the coastline. The interactions of energy conversion devices and the coastal dynamic processes affect the performance of energy extraction, the device safety and the surrounding environment.

Knowledge of coastal waves, currents, sediment transport and morphological evolution is used in various stages of the marine renewable energy engineering project. This includes the assessment of energy density distribution in the nearshore and offshore areas, the site selection of the project, the layout optimization, the prediction of hydrodynamic and morphodynamic responses on both small and large scales, the stress analysis of the device structure induced by wave-current forcing, the prediction and protective measures for local scour and seabed instability around the foundation, and so on.

(5) Coastal dynamics is important for integrated coastal management

China has put forward the strategic thinking of integrated coastal management for the sustainable socio-economic development and eco-environmental protection in coastal zones. The process of coastal management involves frequent and interactive decision-making for a sustainable use, development and protection of coastal areas, their resources and human community, as well as for an enhanced coastal resilience to climate change.

The integrated coastal management will need many efficient practical tools to monitor, understand, evaluate, simulate and predict the rapidly changing coastal environments. Various modern technologies, e. g. , satellite remote sensing, shore-based video imaging system, advanced numerical models, big data and artificial intelligence, have recently been increasing used and shown their robustness in coastal management. The concept of digital coast also emerges to be a promising direction for improving the level of China's coastal management in the coming future.

The developments of coastal management tools are essentially based on a better interpretation of coastal hydrodynamic and morphodynamic processes, the inclusion of more physical and

ecological processes, and the improvements of theoretical or empirical relationships among various dynamic processes. These can support a variety of decision-making procedures including coastal spatial planning, coastal disaster prevention and mitigation, coastal resources utilization and restoration, coastal engineering construction, beach safety warning and coastal science popularization, etc.

Note

aquaculture 水产养殖
Carbon Emission Peak 碳达峰
Carbon Neutrality 碳中和
coastal disaster prevention and mitigation 海岸防灾减灾
coastal engineering 海岸工程
coastal management 海岸管理
coastal protection 海岸保护
coastal resilience 海岸韧性
coastal restoration 海岸修复
coastal resources utilization 海岸资源利用
coastal spatial planning 海岸空间规划
coastal tourism 滨海旅游
cross-sea bridges and tunnels 跨海桥隧
Ecological Civilization 生态文明
energy engineering 能源工程
estuarine regulation 河口治理
fishery engineering 渔业工程
gross marine economic product 海洋经济生产总值
islands and reefs 岛礁
physical model 物理模型
Maritime Power Strategy 海洋强国战略
marine renewable energy 海洋可再生能源
marine ranching 海洋牧场
Maritime Silk Route 海上丝绸之路
maritime transportation 海上交通运输
numerical model 数学模型
Nature-Based Solutions 基于自然的解决方案
oil extraction and mining 采油采矿
port construction 港口建设
reclamation 海涂围垦

science popularization 科学普及

ship repair and shipbuilding 修船造船



Introduction to coastal dynamics 海岸动力学引言



Types and features of coasts 海岸类型与特性

Chapter 2

Wave Theory

2.1 Introduction

Ocean waves are undulations of the sea surface resulting from the transfer of energy. Waves and wave-induced currents are the most important factors in the transportation and deposition of coastal sediment and the morphology evolution in the coastal area. Waves are effective in suspending and moving seabed materials. They can stir and suspend the sediments so that the currents can transport them. Waves shape beaches by transporting sediment from deep waters towards the shore as well as washing and removing fine particles from the shore to deep waters. Many coastal structures have been constructed for different purposes. Jetties and breakwaters have been built to stabilize the sediment and suppress beach erosion. Breakwaters have been built to protect the harbor from energetic waves. For these structures, waves are the main external load. In this chapter, we will learn what waves are and how they behave. We will learn how to describe ocean waves quantitatively and understand the underlying principles.

2.1.1 Causes of Ocean Waves

Generally speaking, most ocean wave energy comes from wind. Waves are created as wind transfers energy to the ocean's surface. Far from shore, in the deep ocean, winds blow over the surface of the water and change the water's surface, first into ripples and then into waves. Once the surface becomes uneven, the wind has an increasing grip on it. The disturbance is propagated by the interactions of disturbing forces (e. g. , wind) and restoring (e. g. , gravity) forces. The water gains energy from the wind due to the friction between the wind and the water surface. Wave motion over most coastal waters has large seasonal variability in response to the changing wind systems in different seasons. The frequency of wind-generated waves ranges from 10^{-2} to 10^2 Hz and the period ranges from 0.01 to 100 seconds.

The tide in the ocean is a kind of long-period wave. Tides are generated by the combined gravitational force exerted on the ocean by the sun and the moon. The tidal frequencies are on the order of 10^{-6} to 10^{-4} Hz and the periods of principle tides are typically around 12 or 24 hours.

Ocean waves can also be caused by earthquakes, volcanic eruptions or large marine landslides. Examples of these kinds of waves are tsunamis and seismic sea waves, of which the frequency is normally less than 10^{-2} Hz.

Ocean waves can be classified according to the disturbing and restoring forces involved in their generation:

(1) Capillary waves: They are small-amplitude waves where the wind is often the disturbing force and surface tension of the water is the restoring force.

(2) Wind waves: Wind is the disturbing force and gravity is the restoring force.

(3) Tides: Gravitational force of the moon and the sun is the disturbing force.

(4) Tsunamis or seismic sea waves: They are caused by earthquakes, volcanic eruptions or large marine landslides.

Ocean waves can also be categorized according to wave periods or frequencies. For the wind-generated waves, the wave height and wave period depend on the wind field. Wind field can be defined in terms of the fetch dimensions, wind duration, and wind speed. Wind fetch is an area of the sea surface over which a wind with a constant direction and speed is blowing. Wind duration is the time that the wind has been in contact with the ocean surface within the wind fetch. The wave height increases as the wind speed, duration, and fetch area increase. Consequently, the nature and intensity of wave action against coastlines vary with the size of the water body as well as wind condition. In this regard, waves can be further divided into a fully developed state of the sea at which the wind has imparted the maximum energy to the waves, and a non-fully developed state of the sea at which the fetch or duration time has limited the amount of wind energy imparted to the waves. Waves within the area of generation are steep and chaotic and have a broad frequency spectrum. Swells are long-crested, sinuous waves that have traveled out of the wind fetch. Swells have a narrow frequency spectrum.

2.1.2 Wave Parameters

In the beginning, the basic wave characteristics, as shown in Fig. 2.1.1, should be defined. All waves have the following properties in common.

Wave surface elevation (η): The instantaneous vertical displacement of wave surface with respect to the still water level, at a certain horizontal location and time.

Wave crest and wave trough: The highest point of the wave surface is called the wave crest, and the lowest point is called the wave trough.

Amplitude (a): The amplitude of a wave is the maximum vertical displacement of a particle at the wave surface from its resting position. In the case of ocean waves, the resting position is at the mean sea level.

Wave height (H): Wave height is the vertical distance between a crest and a nearby trough.

Wave length (L): The horizontal distance between two consecutive crests or two consecutive

troughs is called wave length. It is generally difficult to measure, and is commonly calculated from other wave parameters.

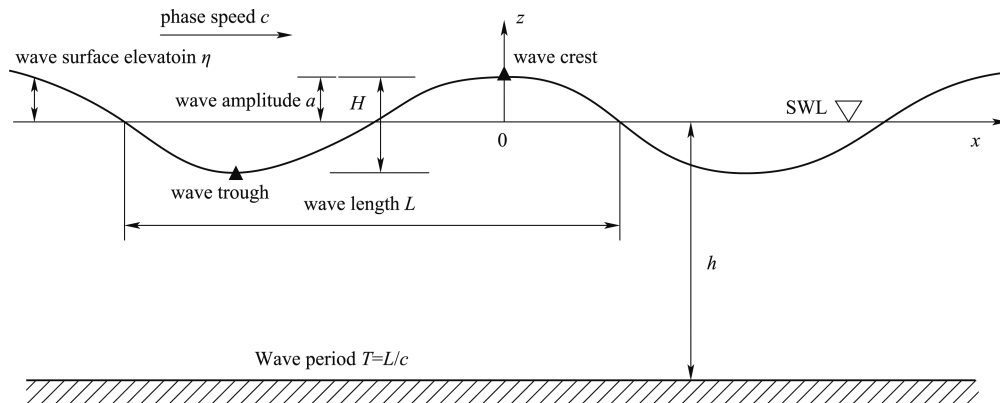


Fig. 2. 1. 1 Schematic diagram of a progressive wave

Wave period (T): Wave period is defined as the time for each crest passing a certain location. In practice, we might measure the duration time of the passage of 11 wave crests and divide it by 10 to obtain wave period. It is noted that wave period generally varies little from deep water to shallow water.

Frequency (f): Frequency is the number of waves to pass a point per unit of time ($f = 1/T$). It is the reciprocal of the period. In general, the lower the frequency, the longer the period, and vice versa.

Wave celerity (c): Wave celerity, also called phase velocity or phase speed, is the rate that a particular phase of wave motion propagates through the medium or the rate that a wave crest propagates through the water. It can be calculated by dividing the wave length by the wave period. It is the propagation speed of an individual wave shape and is expressed as $c = L/T = \omega/k$, where $\omega = 2\pi/T$ is the angular frequency and $k = 2\pi/L$ is the wave number.

Group speed (c_g): Group speed is the speed of the wave train or wave energy, not the speed of an individual wave.

Relative depth (h/L): The relative depth is used to distinguish deep-water, intermediate, and shallow-water waves. Deep water is defined as the water depth of $h/L > 0.5$. In the deep water, wave celerity is not influenced by water depth. This is the case for most wind waves in the deep ocean. Intermediate water is defined as the water depth of $0.05 < h/L < 0.5$. Shallow water is defined as the water depth of $h/L < 0.05$. In the shallow water, wave celerity is mostly controlled by water depth and has less dependence on wave period.

Orthogonal (wave ray): Orthogonal line shows the direction of wave propagation. It is normally perpendicular to the wave crest line.

2.2 Mathematical Description of Wave Motion

2.2.1 Basic Assumptions

Since the sea is very complicated, a simplified theory omitting the most complicated factors is useful for most coastal engineering problems, in which appropriate assumptions should be made first. It is noted that, however, not all assumptions apply to all problems. With respect to different problems, different assumptions should be made to develop suitable wave theories.

In this section, some of the simple mathematical equations for wave motion are introduced. Their limitations, caused by the simplifying assumptions involved, are then indicated.

Here we consider a normally incident train of long, smooth and regular waves, and the smooth wave surface follows a continuous sinuous line. Regular waves belong to a parallel train with a constant wave length. The most important assumptions which are commonly made in wave theories are listed as below :

- (1) The fluid is homogeneous and incompressible while the water density is constant.
- (2) Surface tension can be neglected.
- (3) Coriolis force can be neglected.
- (4) Pressure at the free surface is uniform and constant.
- (5) The fluid is ideal or inviscid and irrotational.
- (6) The wave under consideration does not interact with any other water motion.
- (7) The bed can be described by a horizontal, fixed, impermeable boundary, which implies that the vertical velocity at the bed is zero.
- (8) Wave amplitude is small compared to the depth, and the wave does not change its form with time or space.
- (9) Waves are plane or long crested.

2.2.2 Coordinate Definition

Derivation of wave theories requires the use of a coordinate system. Various systems such as the Cartesian coordinate system or spherical coordinate system can be used depending on the spatial scale of the problem. In this chapter, we will use the Cartesian coordinate system.

As shown in Figure 2.1.1, we consider that waves are inside a two-dimensional vertical water domain with a uniform depth h , and propagating in the horizontal direction with x -axis. z is the vertical coordinate (positive in the upward direction), $z = 0$ and $z = -h$ correspond to the elevations of still water level and seabed, respectively. The free surface of water wave is described by $\eta(x, t)$, where t is time.

2.2.3 Governing Equations

Since the fluid is assumed to be irrotational, potential flow theory can be used to describe the wave motion. The velocity potential $\phi(x, z, t)$ can be related to the flow velocity components u and w in the horizontal (x) and vertical (z) directions by

$$u = \frac{\partial \phi}{\partial x} \quad w = \frac{\partial \phi}{\partial z} \quad (2.2.1)$$

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (2.2.2)$$

Combining Eq. (2.2.1) with Eq. (2.2.2) gives the well-known Laplace's Equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2.2.3)$$

2.2.4 Boundary Conditions

(1) Dynamic free surface boundary condition

For an incompressible fluid in a gravitational field, Bernoulli's full equation at the sea surface is

$$\left(\frac{\partial \phi}{\partial t} \right)_{z=\eta} + \frac{1}{2} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right]_{z=\eta} + \frac{p_0}{\rho} + g\eta = 0 \quad (2.2.4)$$

where p_0 is the atmospheric pressure at the free surface, ρ is the water density, and g is the gravity acceleration.

(2) Kinematical free surface boundary condition

The kinematical boundary condition at the sea surface is given as

$$\frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x} \left(\frac{\partial \phi}{\partial x} \right)_{z=\eta} = \left(\frac{\partial \phi}{\partial z} \right)_{z=\eta} \quad (2.2.5)$$

(3) Impermeable bottom boundary condition

The vertical velocity component at the seabed is zero under impermeable bottom boundary condition:

$$(w)_{z=-h} = \left(\frac{\partial \phi}{\partial z} \right)_{z=-h} = 0 \quad (2.2.6)$$

The Laplace's equation defined in Eq. (2.2.3) and the appropriate boundary conditions in Eqs. (2.2.4)-(2.2.6) provide the basis for the derivation of wave theories.

2.3 Linear Wave Theory

2.3.1 Introduction

The original theory of two-dimensional small waves is attributed to Airy (1841). Airy wave

theory (also referred to as linear wave theory) is based on a linearized description of the propagation of gravity waves at the surface of a homogeneous fluid layer. Most earlier attempts used potential flow theory to describe gravity surface waves.

Although Airy wave theory is very simple, it can provide insight for all periodic wave behaviors that are fairly adequate for many practical applications in coastal and ocean engineering. For the random seas, the Airy wave theory provides a description of wave kinematics and dynamics with a high accuracy. Moreover, several second-order nonlinear properties of surface gravity waves can be estimated using this theory. Wave diffraction, shoaling and refraction, which are important natural wave processes, can also be described by Airy wave theory. This linear theory is often used to get a quick and approximate estimation of the wave characteristics over a large-scale sea area, and is still widely used in many engineering applications nowadays.

2.3.2 Mathematical Formulation

(1) Assumptions

Airy wave theory assumes that the fluid layer has a uniform depth, and that the fluid flow is inviscid, incompressible and irrotational. The most restrictive assumption is that wave amplitude is very small compared to wave length. In general, the Airy wave theory is a linear theory for the propagation of water waves at the surface of a potential flow over a horizontal uniform bottom.

(2) Governing Equations

Since the flow is assumed to be incompressible and irrotational, potential theory can be used. The velocity potential $\phi(x, z, t)$ is related to the flow velocity components u and w in the horizontal (x) and vertical (z) directions by

$$u = \frac{\partial \phi}{\partial x} \quad \text{and} \quad w = \frac{\partial \phi}{\partial z} \quad (2.3.1)$$

According to the mass continuity rule for the incompressible fluid, the velocity potential ϕ should satisfy the well-known Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2.3.2)$$

(3) Boundary Conditions

The boundary conditions of ocean waves at the free surface have non-linear characteristics. Therefore, the general formulation of waves appears to be a complex non-linear problem. In order to simplify the problem, an assumption is made in the linear wave theory that wave amplitude is very small compared to wave length. By doing this, the second-order non-linear terms can be eliminated.

Impermeable bottom boundary condition: The bed boundary is assumed to be impermeable, leading to the kinematic bed boundary condition, defined as

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -h \quad (2.3.3)$$

Kinematic free surface boundary condition: At the free surface, for infinitesimal waves, the vertical acceleration of the free surface has to be equal to the vertical velocity at the free surface:

$$\frac{\partial \eta}{\partial t} = \left(\frac{\partial \phi}{\partial z} \right)_{z=0} \quad (2.3.4)$$

If the free surface elevation $\eta(x, t)$ is known, this would be sufficient to solve the flow problem. However, the surface elevation is also an unknown variable, for which an additional boundary condition is required. This is provided by Bernoulli's equation for an unsteady potential flow.

Dynamic free surface boundary condition: The linear wave theory neglects non-linear terms involving the product of two unknown variables. For an incompressible fluid in a gravitational field, Bernoulli's equation is reduced to

$$\left(\frac{\partial \phi}{\partial t} \right)_{z=\eta} + \frac{p_0}{\rho} + g\eta = 0 \quad (2.3.5)$$

The atmospheric pressure at the free surface, p_0 , is assumed to be constant. This constant atmospheric pressure is taken as zero without loss of generality. Then Bernoulli's equation will be further reduced to

$$\left(\frac{\partial \phi}{\partial t} \right)_{z=0} + g\eta = 0 \quad (2.3.6)$$

The surface elevation may be expressed as

$$\eta = -\frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right)_{z=\eta=0} \quad (2.3.7)$$

From the Eqs. (2.3.4) and (2.3.7) we get

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \quad (\text{at } z=0) \quad (2.3.8)$$

Note that because a linear wave is assumed for both free surface boundary conditions described above, the values of ϕ and $\partial\phi/\partial z$ are defined at the still water surface ($z=0$).

With these assumptions, the surface elevation $\eta(x, t)$ of a single wave traveling in the x direction is a function of the horizontal position x and time t :

$$\eta(x, t) = a \cos(kx - \omega t) \quad (2.3.9)$$

where a is wave amplitude, k is wave number related to wave length L :

$$k = \frac{2\pi}{L} \quad (2.3.10)$$

and ω is the angular frequency, related to wave period T :

$$\omega = \frac{2\pi}{T} = 2\pi f \quad (2.3.11)$$

The wave celerity c of a propagating wave is then expressed as

$$c = \frac{\omega}{k} = \frac{L}{T} \quad (2.3.12)$$

2.3.3 Analytical Solution

(1) Velocity potential

The associated velocity potential, satisfying the Laplace equation in the fluid interior and the boundary conditions described above, can be derived as

$$\phi = \frac{ag}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin(kx - \omega t) \quad (2.3.13)$$

or

$$\phi = \frac{a\omega}{k} \frac{\cosh k(h+z)}{\sinh kh} \sin(kx - \omega t) \quad (2.3.14)$$

where \sinh and \cosh are the hyperbolic sine and hyperbolic cosine functions, respectively.

(2) Dispersion relation

The dispersion relation for the linear wave is given by

$$\omega^2 = gk \tanh(kh) \quad (2.3.15)$$

where \tanh is the hyperbolic tangent function.

From the above dispersion relation, we find that angular frequency ω and wave number k are coupled. ω and k must satisfy the dispersion relation while wave amplitude a can be chosen freely in Airy wave theory. It is also found that gravity surface waves exhibit frequency dispersion, indicating that each wave has its own frequency and celerity. In other words, waves of different frequencies travel at different speeds and waves become sorted.

Two approximations are useful in practice, including the deep-water approximation and the shallow-water approximation. Deep-water approximation is valid if water depth h is much larger than wave length. In this case, $h \gg L$, $kh \gg 1$, and $\tanh(kh) = 1$. Shallow-water approximation is valid if water depth is much smaller than wave length. In this case, $h \ll L$, $kh \ll 1$, and $\tanh(kh) = kh$. For these two special cases, the dispersion relation can be simplified to:

Deep-water dispersion relation:

$$\omega^2 = gk \quad h/L > 0.5 \quad (2.3.16)$$

Shallow-water dispersion relation:

$$\omega^2 = gk^2 h \quad h/L < 0.05 \quad (2.3.17)$$

(3) Wave celerity

Wave celerity (or phase velocity, phase speed) can be easily obtained from the dispersion relation:

$$c = \frac{\omega}{k} = \frac{gT}{2\pi} \tanh \frac{2\pi h}{L} \quad (2.3.18)$$

Under deep-water and shallow-water approximations, wave celerity can be simplified to:

Deep-water wave celerity:

$$c = \frac{gT}{2\pi} \quad (2.3.19)$$

Shallow-water wave celerity :

$$c = \sqrt{gh} \quad (2.3.20)$$

In deep water, wave celerity depends on wave length or wave frequency. Longer waves travel faster. Thus, deep-water waves are dispersive. In shallow water, however, wave celerity only depends on water depth. All waves at a given shallow water depth travel at the same speed.

(4) Particle velocity

The velocities of a water particle are expressed as

$$u = \frac{\partial \phi}{\partial x} = \frac{\pi H}{T} \frac{\cosh[k(z+h)]}{\sinh(kh)} \cos(kx - \omega t) \quad (2.3.21)$$

$$w = \frac{\partial \phi}{\partial z} = \frac{\pi H}{T} \frac{\sinh[k(z+h)]}{\sinh(kh)} \sin(kx - \omega t) \quad (2.3.22)$$

The above equations demonstrate that water particle velocities under the linear waves are maximum at the surface and decrease in magnitude with depth. The maximum horizontal and vertical velocities (occurring at the water surface) are denoted as u_{\max} and w_{\max} , respectively.

The directions of particle motion are related to the motion of the water surface (Fig. 2.3.1). At the wave crest, the particle motion is horizontal and in the direction of wave propagation. At the wave trough, the direction of particle motion is reversed but the velocity is of the same magnitude. Vertical velocities reach their maxima when the surface particle crosses the still water level.

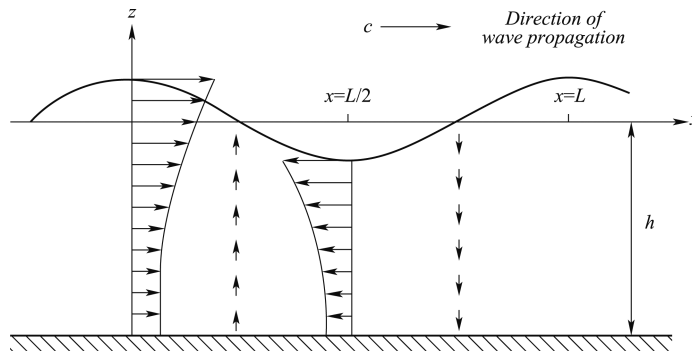


Fig. 2.3.1 Water particle velocity distribution

(5) Particle path

For a given water particle, the variation of its position with time can be described by

$$\frac{dx}{dt} = \frac{\partial \phi[x(t), z(t), t]}{\partial x} \quad (2.3.23)$$

$$\frac{dz}{dt} = \frac{\partial \phi[x(t), z(t), t]}{\partial z} \quad (2.3.24)$$

The position for a given water particle can be described by the sum of its mean position (x_0 , z_0) and its horizontal and vertical displacements, $\xi(t)$ and $\zeta(t)$:

$$x(t) = x_0 + \xi(t) \quad z(t) = z_0 + \zeta(t) \quad (2.3.25)$$

Substituting Eq. (2.3.25) into Eq. (2.3.23) and Eq. (2.3.24), expanding them in a Taylor series and neglecting the higher-order terms, we have

$$\frac{d\xi}{dt} = a\omega \frac{\cosh k(h+z_0)}{\sinh kh} \cos(kx_0 - \omega t) \quad (2.3.26)$$

$$\frac{d\zeta}{dt} = a\omega \frac{\sinh k(h+z_0)}{\sinh kh} \sin(kx_0 - \omega t) \quad (2.3.27)$$

Integrating the above equations, we obtain the particle displacements in the horizontal and vertical directions, respectively:

$$\xi = x - x_0 = -a \frac{\cosh k(h+z_0)}{\sinh kh} \sin(kx_0 - \omega t) \quad (2.3.28)$$

$$\zeta = z - z_0 = a \frac{\sinh k(h+z_0)}{\sinh kh} \cos(kx_0 - \omega t) \quad (2.3.29)$$

Eliminating t from the above two equations, the particle trajectory can be described by

$$\frac{(x-x_0)^2}{\left[\frac{H}{2} \frac{\cosh k(h+z_0)}{\sinh kh}\right]^2} + \frac{(z-z_0)^2}{\left[\frac{H}{2} \frac{\sinh k(h+z_0)}{\sinh kh}\right]^2} = 1 \quad (2.3.30)$$

It is evident that the particle orbit is closed and elliptical, as shown in Fig. 2.3.2. Therefore, there is no net mass transport over a wave period in the Airy wave theory.

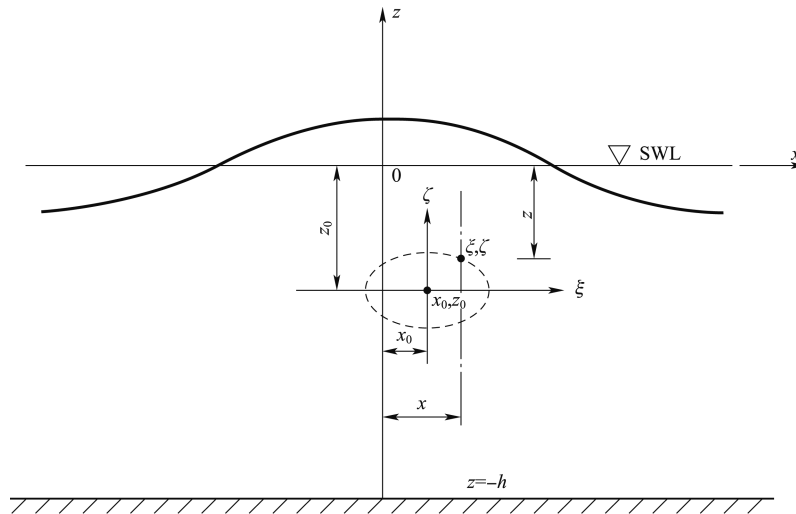


Fig. 2.3.2 Water particle path

If the horizontal and vertical semi-axes are expressed respectively by a_p and b_p , we can get

$$\frac{(x-x_0)^2}{a_p^2} + \frac{(z-z_0)^2}{b_p^2} = 1 \quad (2.3.31)$$

in which

$$a_p = \frac{H}{2} \frac{\cosh[k(z_0+h)]}{\sinh(kh)} \quad (2.3.32)$$

$$b_p = \frac{H}{2} \frac{\sinh[k(z_0 + h)]}{\sinh(kh)} \quad (2.3.33)$$

For shallow water ($h/L < 0.05$), the horizontal motion varies little over the depth. The elliptical paths followed by the water particles flatten to horizontal lines, particularly at the bottom where no vertical motion is allowed (as shown in Fig. 2.3.3). It is evident that the shallower the water, the flatter the ellipse.

For the case of deep water ($h/L > 0.5$), we have

$$a_p = \frac{H}{2} \frac{\cosh[k(z_0 + h)]}{\sinh kh} = \frac{H}{2} \frac{e^{k(z_0+h)} + e^{-k(z_0+h)}}{e^{kh} - e^{-kh}} = \frac{H}{2} e^{kz_0} \quad (2.3.34)$$

$$b_p = \frac{H}{2} e^{kz_0} \quad (2.3.35)$$

Fig. 2.3.4 shows the water particle paths under waves in deep water. It can be seen that the orbits are approximately circular. At the surface $a_p = b_p = H/2$, and at the bottom $a_p = b_p = 0$.

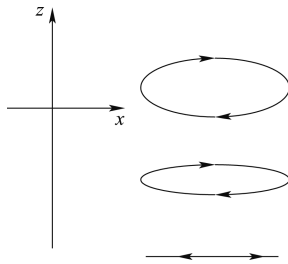


Fig. 2.3.3 Water particle paths under waves in shallow water

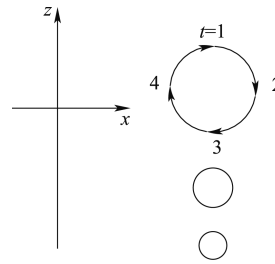


Fig. 2.3.4 Water particle paths under waves in deep water

(6) Wave pressure

Based on Bernoulli's equation and neglecting the second-order terms, the water pressure beneath surface waves is expressed as

$$p = -\rho \frac{\partial \phi}{\partial t} - \rho g z \quad (2.3.36)$$

The second term on the right-hand side of the above equation represents the hydrostatic pressure. The first term denotes the dynamic pressure induced by waves, which can be further written as

$$-\rho \frac{\partial \phi}{\partial t} = \rho g \frac{\cosh k(h+z)}{\cosh kh} \eta \quad (2.3.37)$$

We further define a pressure response factor as

$$K_p = \frac{\cosh k(h+z)}{\cosh kh} = \frac{\cosh \left[kh \left(1 + \frac{z}{h} \right) \right]}{\cosh kh} \quad (2.3.38)$$

It can be seen that K_p decreases as water depth increases. The distribution of wave dynamic pressure over water depth is shown in Fig. 2.3.5.

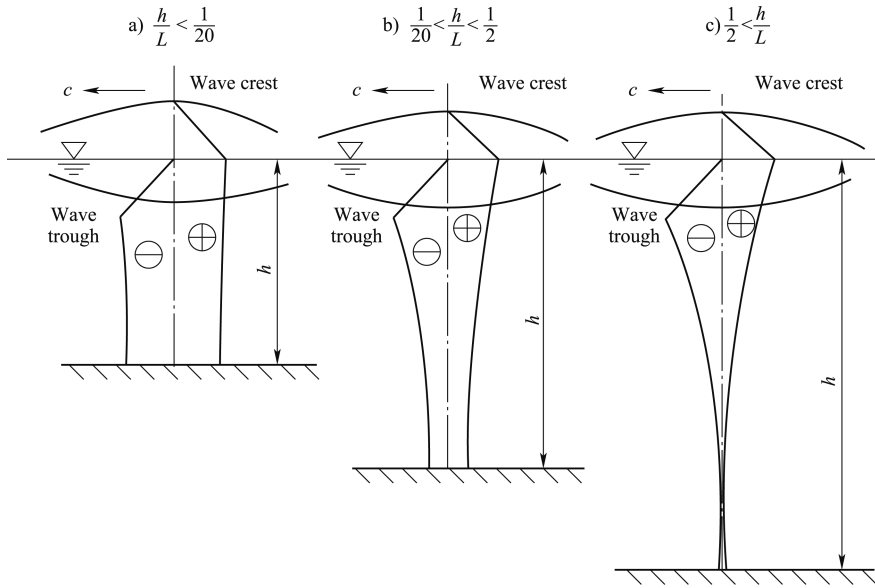


Fig. 2.3.5 Distributions of wave dynamic pressures in cases of shallow (left), intermediate (middle) and deep (right) water depths

(7) Wave energy

Wave energy E and its transfer with wave trains are of great importance in coastal dynamics. The potential wave energy (E_p) per unit wave crest width over one wave length is given by

$$E_p = \int_0^L \int_{-h}^{\eta} \rho g z dz dx = \frac{1}{16} \rho g H^2 L \quad (2.3.39)$$

The kinetic wave energy per unit wave crest width over one wave length, E_k , is given by

$$E_k = \frac{1}{2} \int_0^L \int_{-h}^{\eta} (\overset{2}{u} + \overset{2}{w}) \rho dz dx = \frac{1}{16} \rho g H^2 L \quad (2.3.40)$$

As can be seen, the kinetic and potential energy of linear waves are equal. The total energy per unit wave crest width over one wave length, E , can then be expressed as

$$E = E_k + E_p = \frac{1}{8} \rho g H^2 L \quad (2.3.41)$$

Note that wave energy is proportional to the square of wave height. In water waves, the most used energy measure is the mean wave energy density per unit horizontal area. It is the sum of kinetic and potential energy density, integrated over the depth and the wave phase. The total wave energy density per unit surface area is given as

$$\bar{E} = \bar{E}_k + \bar{E}_p = \frac{1}{8} \rho g H^2 = 2\bar{E}_p = 2\bar{E}_k \quad (2.3.42)$$

where the over-bar denotes the mean value (which in the present case of periodic waves can be taken either as a time average or an average over one wave length in space).

Taking a vertical cross-section with unit crest width, we can calculate the energy flux passing through this section per unit as time:

$$W = \int_{-h}^{\eta} p u dz \approx \int_{-h}^0 p \frac{\partial \phi}{\partial x} dz \quad (2.3.43)$$

Substituting Eqs. (2.3.14) and (2.3.36) into Eq. (2.3.43) yields:

$$W = \rho g c n \eta^2 + \frac{\rho g c n}{k} \left(\coth kh - \frac{1}{\sinh kh} \right) \quad (2.3.44)$$

The mean energy flux over one wave period is:

$$\bar{W} = \frac{1}{T} \int_0^T \int_{-h}^{\eta} p u dz dt \approx \frac{1}{T} \int_0^T \int_{-h}^0 p u dz dt = \bar{E} c_g \quad (2.3.45)$$

The above equation indicates that the mean energy flux over one wave period is the product of wave energy density and the transfer velocity of wave energy (c_g , also called the wave group velocity discussed in the next section).

(8) Wave group

In reality, ocean surface waves are a combination of many individual waves. Even in a regular swell, there are many wave components but they tend to be grouped with respect to the different group velocities. Group velocity c_g is a fundamental concept in wave theory. First, it is the propagation velocity of wave energy as mentioned above. Second, it is the velocity at which a wave group travels.

We shall see how simple waves with close wave heights, wave lengths and velocities combine to form a wave group. This phenomenon is common and can be easily observed in the real ocean. Considering two simple harmonic waves:

$$\eta_1 = a \cos k(x - ct) \quad (2.3.46)$$

$$\eta_2 = a \cos k'(x - c't) \quad (2.3.47)$$

They have the same wave amplitude and their wave numbers and velocities are almost equal:

$$k \approx k' \quad (2.3.48)$$

$$c \approx c' \quad (2.3.49)$$

The combination of these two waves gives

$$\begin{aligned} \eta &= a \cos k(x - ct) + a \cos k'(x - c't) \\ &= 2a \cos \left(\frac{k - k'}{2} x - \frac{kc - k'c'}{2} t \right) \cos \left(\frac{k + k'}{2} x - \frac{kc + k'c'}{2} t \right) \end{aligned} \quad (2.3.50)$$

Therefore, the resultant wave can be considered as a single sine wave with the time-varying amplitude given as $a \cos \left(\frac{k - k'}{2} x - \frac{kc - k'c'}{2} t \right)$.

This time-varying amplitude is also a progressive wave with the wave length, period and celerity given by $\frac{4\pi}{k - k'}$, $\frac{4\pi}{kc - k'c'}$ and $\frac{kc - k'c'}{k - k'}$.

Fig. 2.3.6 shows the resultant wave at a specific time. The dashed lines in the figure indicate the envelope of surface elevation.

Here the propagation speed of the wave train is interpreted as the group velocity:

$$c_g = \frac{kc - k'c'}{k - k'} = \frac{\Delta(kc)}{\Delta k} \quad (2.3.51)$$

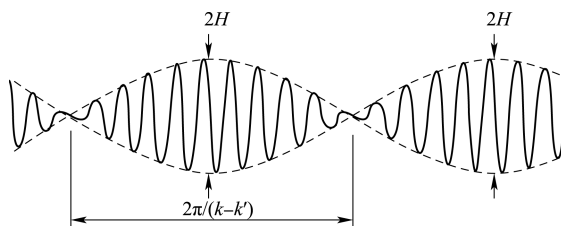


Fig. 2. 3. 6 Combination of two waves with the same amplitude but with slightly different wave numbers and phase velocities

$$c_g = \frac{d(kc)}{dk} = c + k \frac{dc}{dk} = c - L \frac{dc}{dL} \tag{2. 3. 52}$$

Substituting the expression of wave celerity (c) into Eq. (2. 3. 52) yields the following relationships :

$$c_g = \frac{1}{2} \left[1 + \frac{4\pi h}{L} \frac{1}{\sinh(4\pi h/L)} \right] c = nc \tag{2. 3. 53}$$

$$n = \frac{1}{2} \left[1 + \frac{4\pi h}{L} \frac{1}{\sinh(4\pi h/L)} \right] \tag{2. 3. 54}$$

In fact, the definition of group velocity in two dimensions is :

$$c_g = \frac{\partial \omega}{\partial k} \tag{2. 3. 55}$$

Under deep-water and shallow-water approximations, the group velocity can be approximated by :

Group velocity in deep water :

$$c_g = \frac{g}{2\omega} = \frac{c}{2} \tag{2. 3. 56}$$

Group velocity in shallow water :

$$c_g = \sqrt{gh} = c \tag{2. 3. 57}$$

2. 4 Stokes Wave Theory

Wave theories can be used to predict the surface shape and kinematics of waves in deep or shallow water. As mentioned above, the Airy wave theory provides adequate approximations of wave motion for many engineering applications. However, when waves become relatively large or they are propagating in very shallow water, the small-amplitude theory is no longer valid and the non-linear terms are no longer ignorable. In such cases, higher-order wave theories, such as Stokes wave theory, cnoidal wave theory and solitary wave theory may be more appropriate.

In Airy wave theory, we derived the properties of an ocean surface wave based on the assumption that the wave amplitude is infinitely small $ka = o(0)$. The linear theory was extended by Stokes (1847) for non-linear waves. The key of the Stokes wave theory is that if waves are small ($ka \ll -1$) but not infinitely small, wave properties can be expanded in a power series of ka .

In general, the perturbation expansion for velocity potential ϕ can be represented in forms of a power series expansion of some small parameter ε :